Unit Planner: Introduction to Triangle Trigonometry
Math Analysis II (H) (HS)

Friday, August 25, 2017, 10:26AM

Standards & Benchmarks

NJSLS-S: Science and Engineering Practices

NJSLS-S: 9-12

Practice 2. Developing and using models

Modeling in 9–12 builds on K–8 experiences and progresses to using, synthesizing, and developing models to predict and show relationships among variables between systems and their components in the natural and designed worlds.

Design a test of a model to ascertain its reliability.

Develop a complex model that allows for manipulation and testing of a proposed process or system.

Develop and/or use a model (including mathematical and computational) to generate data to support explanations, predict phenomena, analyze systems, and/or solve problems.

Practice 3. Planning and carrying out investigations

Planning and carrying out investigations in 9-12 builds on K-8 experiences and progresses to include investigations that provide evidence for and test conceptual, mathematical, physical, and empirical models.

Select appropriate tools to collect, record, analyze, and evaluate data.

Practice 4. Analyzing and interpreting data

Analyzing data in 9–12 builds on K–8 experiences and progresses to introducing more detailed statistical analysis, the comparison of data sets for consistency, and the use of models to generate and analyze data.

Analyze data using tools, technologies, and/or models (e.g., computational, mathematical) in order to make valid and reliable scientific claims or determine an optimal design solution.

Consider limitations of data analysis (e.g., measurement error, sample selection) when analyzing and interpreting data.

NJ: 2016 SLS: English Language Arts

NJ: Grades 9-10

Capacities of the Literate Individual

Students Who are College and Career Ready in Reading, Writing, Speaking, Listening, & Language

They demonstrate independence.

They build strong content knowledge.

They value evidence.

They use technology and digital media strategically and capably.

Writing

NJSLSA.W2 Write informative/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content.

W.9-10.2d. Use precise language and domain-specific vocabulary to manage the complexity of the topic.

NJSLSA.W5 Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach.

W.9-10.5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, trying a new approach, or consulting a style manual (such as MLA or APA Style), focusing on addressing what is most significant for a specific purpose and audience. (Editing for conventions should demonstrate command of Language standards 1–3 up to and including grades 9–10 on page 55.)

NJSLSA.W6 Use technology, including the Internet, to produce and publish writing and to interact and collaborate with others.

W.9-10.6. Use technology, including the Internet, to produce, share,
and update writing products, taking advantage of technology's capacity to link to other information and to display information flexibly and dynamically.

Speaking and Listening

Comprehension and Collaboration

NJSLSA.SL1 Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.

SL.9-10.1a. Come to discussions prepared, having read and researched material under study; explicitly draw on that preparation by referring to evidence from texts and other research on the topic or issue to stimulate a thoughtful, well-reasoned exchange of ideas.

SL.9-10.1d. Respond thoughtfully to various perspectives, summarize points of agreement and disagreement, and justify own views. Make new connections in light of the evidence and reasoning presented.

NJSLSA.SL5 Make strategic use of digital media and visual displays of data to express information and enhance understanding of presentations.

SL.9-10.5. Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance findings, reasoning, and evidence and to add interest.

Language

Conventions of Standard English

NJSLSA.L1 Demonstrate command of the conventions of standard English grammar and usage when writing or speaking.

L.9-10.1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking.

L.9-10.1b. Use various types of phrases (noun, verb, adjectival, adverbial, participial, prepositional, absolute) and clauses (independent, dependent; noun, relative, adverbial) to convey specific meanings and add variety and interest to writing or presentations.

NJSLSA.L2 Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing.

L.9-10.2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing.

L.9-10.2c. Spell correctly.

NJ: 2016 SLS: Mathematics

NJ: HS: Functions

Trigonometric Functions

HSF-TF.A. Extend the domain of trigonometric functions using the unit circle.

HSF-TF.A.3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for p/3, p/4 and p/6, and use the unit circle to express the values of sine, cosines, and tangent for x, p + x, and 2p – x in terms of their values for x, where x is any real number.

HSF-TF.B. Model periodic phenomena with trigonometric functions.

HSF-TF.B.5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

HSF-TF.B.7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Mathematical Practice

MP.The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

MP.2. Reason abstractly and quantitatively.

MP.3. Construct viable arguments and critique the reasoning of others.

MP.4. Model with mathematics.

MP.5. Use appropriate tools strategically.

MP.6. Attend to precision.

NJ: HS: Geometry

Similarity, Right Triangles, & Trigonometry

HSG-SRT.B. Prove theorems involving similarity

HSG-SRT.B.5. Use triangle congruence and similarity criteria to solve problems and to prove relationships in geometric figures.

HSG-SRT.C. Define trigonometric ratios and solve problems involving right triangles

HSG-SRT.C.6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

HSG-SRT.C.7. Explain and use the relationship between the sine and cosine of complementary angles.

HSG-SRT.C.8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

HSG-SRT.D. Apply trigonometry to general triangles

HSG-SRT.D.10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.

HSG-SRT.D.11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Access the interactive version of the NGSS here

Essential Questions

Justify the distance to the sun.
How do you make conclusions based on patterns you observe?
What does it mean to be similar?
How can it be possible to have one case that leads to one, two or no possible outcome?
Where is triangle trigonometry used in the real world?
What connections could be made between math and other sciences using trigonometry?
How do you use deductive reasoning to draw conclusions?
Is the calculator always right?
How do the definitions of trigonometry functions lead to the various other identities and formulas that are discovered in this unit?

Content

Pythagorean Theorem
Angle of Elevation
Angle of Depression
Right Triangle Trigonometric Functions
Reciprocal Trigonometric Functions
Special Right Triangles
Unit Circle
Inverse Trigonometric Functions
Law of Sines
Law of Cosines

Skills

1. Determine unknown lengths and distances using angles and projections.
2. Finding side lengths of right triangles using trigonometric functions.
3. Finding angles of triangles using inverse trigonometric functions.
4. Apply trigonometric ratios to real-world angles of elevation and depression.
5. Finding trigonometric functions of angles in a 30-60-90 right triangle and 45-45-90 right triangle without using a calculator.
6. Finding inverse trigonometric functions of special ratios leading to angles of 30,45,or 60 degrees without using a calculator.
7. Solving for side lengths and angles using law of sines.
8. Prove the law of sines.
9. Determining when we have 0, 1, or 2 triangles in the law of sines ambiguous case.
10. Solving for side lengths and angles using the law of cosines.
11. Prove the law of cosines.
12. Determining when law of sines or law of cosines should be used.

Assessment

Thumbs Up-Side-Down (daily)
Summative: Imported
Thumbs Up-Side-Down serves as a student self-assessment, as well as, a tool for me to accurately decipher which students are grasping particular tasks and their corresponding confidence level. With this, I get to see the individual response, in addition to the overall class response to decide if lesson modification is necessary to the daily/weekly or unit lesson. Thumbs Up-Side-Down serves as a student self-assessment, as well as, a tool for me to accurately decipher which students are grasping particular tasks and their corresponding confidence level. With this, I get to see the individual response, in addition to the overall class response to decide if lesson modification is necessary to the daily/weekly or unit lesson.
Math Portfolio (constant)
Summative: Other: Student Portfolio
Math Portfolio (constant)
Student performance portfolio of tasks include all work that students have completed throughout the course ranging from assessment (including projects), communication, (if possible) retakes of assessments to demonstrate true understanding of concept as it pertains to various levels of application through written and verbal explanation. This will provide a collection of the progress of the student, as well as, provide an organized collection of their work for them to reflect upon at a later date. Meetings with teacher regarding this portfolio can be scheduled.
Graphic Organizer
Imported
Graphic Organizer: Index Card (for use in pop assessment)
Triangle Trig. Assessment
Summative: Test: Written
Triangle Trigonometry Post-test.docx
Triangle Trigonometry Pre-Test
Test: Common
SGO - Triangle Trigonometry Pre-test.docx
Telescopic Trigonometry Project
Performance: Lab Assignment
This collaborative project will contain a benchmark, as well as, a rubric and description that will be given to students and used when grading the product.
Telescopic Trigonometry.docx
End of Unit Group Project

Project: Technology
Students will have a choice to work individually, in pairs or groups of 3 in this in-class project that will provide a summary of what they have learned (last column of KWL chart).

Students will present their final creations to their classmates.

Project_End of the unit.docx
Student-created assessment
Formative: Test: Written
White Board Review
Performance: Skill Demonstration
informal assessment of student work and solutions

*teacher observation of skill demonstration
Research
Oral: Discussion
Research: Students will work independently and cooperatively to informally research various topics from sources of their choosing and determine the basic foundation of understanding in order to start applying to numerical and real-world scenarios. .

Activities

Prior to beginning this unit, a pre-test was given to determine prior knowledge and modify the unit accordingly

Lesson One:
1. Students will be asked to perform an internet search (completed on individual/class laptops, iPads, smart phones, class SmartBoard, etc.) to determine the exact distance to the sun. Once determined, students will work through a short think-pair-share activity to gather thoughts and share their views on the topic. Class discussion will begin once cooperative groups begin to share their thoughts on their findings.
2. Once complete, students will be introduced to the use of trigonometry in the real-world to determine various distances. A brief historical connection to the use of trigonometry by Egyptians with sundials around 1500 B.C. will set a timeline as to the long-term use of trigonometry in the real-world. Before beginning the activity, students will be given a KWL chart to partially fill out. The first two columns will be completed to determine prior knowledge and what topics students are interested to learn about. KWL charts will be collected for teaching viewing and returned at the end of the unit to reflect and complete.
3. Since the majority of students will show prior knowledge of the Pythagorean Theorem, the discussion of right triangles and their components will occur between cooperative groups to create a class list of characteristics. Students will be given a variety of proofs to justify the Pythagorean theorem that are visual, hands-on and auditory. They will work individually and choose one of the proofs to then show to a classmate that worked on a different proof. Students will work in a round-robin style until all students have seen all proofs. Proofs will include the use Geoboards, multiple videos, as well as, the mathematical algebraic and geometric justifications. Students will discuss other methods to prove the Pythagorean Theorem.

Resources/Optional

http://www.teachertube.com/video/pythagorean-theorem-water-demo-323823
KWL Chart.docx
Similar Right Triangles Introduction.docx
https://www.youtube.com/watch?v=uOTs2ck1_jU
Telescopic Trigonometry.docx
Law of Sine and Cosine mix.pdf
Trigonometric Functions Exercises and Solutions.rtf
Law of Sine and Cosine Study Guide.docx
Triangle Trig Course Unit Resource Student Copy.docx
Project_End of the unit.docx
Telescopic Trigonometry.docx
Triangle Trigonometry Post-test.docx
SGO - Triangle Trigonometry Pre-test.docx
law of sines and cosines mix word problems.pdf
LawofCosines.pdf
LawofSines.pdf
AnglesDepressionElevation.pdf
DegreeConversions.pdf
Solving for “x” with Trigonometric Functions.pdf
Trigonometric Values and Triangles.pdf
Finding Trig Values of Triangles with Given Side Lengths.pdf
Values of Basic Trigonometric Functions.pdf
More Review.doc
extra credit.docx
Triangle Trig mini quiz.docx

Lesson Two:

1. Students will be given a copy of the Similar Right Triangles Intro Worksheet to demonstrate use of a protractor and ruler, as well as, discover the common ratios of each trigonometric function. *Modification: Students may use the Geoboard in addition to the Visual Aid provided for this activity. S*tudents will take on the mission to determine the distance to the sun. Students will work in their respective rows and columns to determine the various angles and side lengths in order to compare the data to the ratio.

2.Students will have a choice between using a Geoboard or measuring a new set of similar triangles and finding their trigonometric ratios. The discussion of experimental versus theoretical values will be included as there may be sources of error from student measurements. Students will compare their self-creations and explain their processes to one another.

Lesson Three:

1. *Research:* Students will work independently and cooperatively to informally research various topics from sources of their choosing and determine the basic foundation of understanding in order to start applying to numerical and real-world scenarios. Students will use prior knowledge to create their own problems to give to their peers to solve. Once complete, discussion of various techniques and methods will occur to ensure all individuals get an introduction to all problems.

2. *Discovery discussion based on research:* Students will include an interesting fact/example they researched in reliable sources (or original real-world examples created by student) on class white board to summarize key concepts of their research. Students will work cooperatively to share ideas, asks questions and demonstrate what they gathered through informal conversion and/or manual calculations on class white boards.

Lesson Four:

1. Students will use a course unit resource created by the teacher to practice daily activities starting with a do-now and continuing with the introduction of a new concept that will use the prior knowledge to solve accurately.

Do Now: Writing Task -> Critical Thinking Composition: How does the role of special right triangle correlate to inverse trig definitions? Provide a visual example with mathematical calculations to justify your reasoning. Concise comparison sentence: Determine the purpose of trigonometric functions as compared to the inverse trigonometric functions to determine their specific application.

Once complete, put at least one component of the homework on the class white board to assess your introductory knowledge & corresponding skills.

2. Through teacher monitoring, students will work on additional practice individually or cooperatively through their selected method. Methods will include seated work in their notebook, seated work with a personal white board, engagement on the class white board, checking the work of their cooperative partners and/or

verbally with their cooperative partner and/or the entire class. Based on student preference, they will have a choice as to how they wish to participate whether it is at the class white board, at their desks with individual white boards or working in their notebook with verbal additions to the class discussion. When at the class white board, students are expected to check their classmates work and determine if their are any errors and then cooperatively come to an accurate conclusion. Additional practice and solutions are included in the course unit resource for students to implement when studying for formal assessments. This self-check allows students the opportunity to be responsible and accountable for their continual learning both in and out of the classroom.
3. Using previous knowledge, students work together to determine how they could draw further conclusions about new areas. A combination of lecture and discussion is used to show the progression of the topic and represent its path in the mathematical timeline.

Lesson Five:
1. *Think-group-share activity:* Students will work in pairs to discuss the various topics in class and be able to ask each other questions or formulate ideas for solutions to then share with classmates during open discussion.
2. *Cooperative Practice:* Students will use trigonometry definitions to prove common trigonometric identities, such as the reciprocal identities, the cofunction identities, and the Pythagorean identities.Students will solve triangles using right triangle trigonometry definitions and inverse trigonometric functions. In specific, students will be given a double-sided worksheet of problems (found in course unit resource) and cooperatively each partner will choose to complete the even or odd problems on the first page. Once complete, students will move around the room to find someone who did the same problems to check their procedures and solutions. Then students will continue to do their selected problems on the second page. Once complete, students will find classmates who did not complete the same problems to check their work. It is crucial that the students do not complete these problems, instead they should be critiquing their classmates work.
3. *Telescopic trigonometry project* will serve as an assessment of student understanding. Students will choose a partner and create their own telescopes to hypothesize distances based using trigonometric ratios. They will then choose a variety of objects to determine the angles of elevation and depression. A classroom model will be demonstrated to discuss the procedures for students to choose larger scales that would be harder to measure (such as height of a tree or a building). The theoretical and experimental trigonometric finding will be found and compared. Causes for error will be discussed and determine the final findings.

Lesson Six:
Special Right Triangles

Through a think-pair-share, students will worker cooperatively to determine prior knowledge of the special right triangles learned in previous year. To organize findings, students will complete a chart finding all of the trig function values of 30, 45, and 60 degrees using special right triangles. Students will have a choice if they would like to create a graphic organizer of the formulas and corresponding data on an index card. Students will also determine inverse trigonometric functions of ratios that come from special right triangles. Once cooperative work is complete, the class will create a universal chart with corresponding visuals. This will later help students with generating terminal points of the unit circle.

Lesson Seven:

1. *Bases*

Students from the Computer Science Academy will help introduce discussion by integrating their knowledge of bases from programming to the commonly used base of ten. The transition to Babylonian base of 60 will lead to the idea of degree decimal and degree-minute-second representation, as well as, the concept of revolution through visuals (clock, unit circle, a ballerina's pirouette) for the dimensional analysis between radians and degrees (and vice versa).

2. Students will derive the relationship between angles and radians by using the definition of a radian and their knowledge that circumference of a circle = 2(pi)r. Students will convert between degrees, radians, and degrees-minutes-seconds form both with and without a calculator. Discussion about ways in which we could prove that triangles are congruent will occur. This conversation will lead into an explanation of law of sines and law of cosines.

Lesson Eight:

1. Students will be given multiple right triangles (of different sizes) to choose from. The altitude shown to recall prior geometric knowledge of congruent and similar triangles. Visually, students will be shown images of the other possible cases besides one unique cases with connections to a swing set and swinging doors to show the concepts of one side moving to be too short, long enough for two true statements or only one unique solutions. Students will work in cooperative pairs through various problems and discuss findings as a class. Problems will be completed on the class white board and checked by peers to ensure accuracy.

2. *Swing example*

Students will learn law of sines and partially derive this formula using their knowledge of right triangle trigonometry. Students will use law of sines and law of cosines to solve both pure and real world triangle problems. Students will explain why the SSA case does not determine a specific triangle, and they will determine whether an SSA case refers to a situation with 0,1,or 2 triangles.

Lesson Nine:

1. Students will be prompted with a think-pair-share activity that will require them to recall prior geometric

knowledge of congruent and similar triangles (for ex: SSS, HL, SAS,..etc.) Once discussion occurs, a short demonstration of the two scenarios where trigonometry is used for right triangles will allow students to see the connection where 'AAS' and 'SSA' can hold true unlike they previously thought. Discussion about unique solutions versus more than one solution, infinite solutions or no solutions will occur to transition students to oblique triangles. Students will be asked what oblique means and if needed, will be provided a visual aid to help. Teacher will demonstrate the breakdown and proof of the law of sines and then lead to visual examples of 'AAS' concrete and real-world applications.
2. Students will work in groups of 3 to create their own triangles and assess their partners by switching and then checking each others work. Before switching, students will ensure their problem can be solved. These designs will help create a student-created assessment.
Students will be given their KWL charts to complete the last column as to what they known and reflect upon their previous responses. Discussions and reflections will occur.
To demonstrate the final column of their KWL chart, students will work in pairs or groups of 3 in an in-class project presentation.

Lesson Ten:
*1.*Recap Session: After a Do-Now, students will write details about topics on the class white board to create a class graphic organizer summary of topics and materials covered. Students will be surveyed about what topics they would like more practice on to determine the lesson accordingly.
2.White Board Activity (Unit Review)
Students will work on a variety of problems individually and cooperatively. Solutions will be checked when students raise their work and solutions on the individual white boards up for me to see and assess quickly. A variety of problems will be given to determine the level of understanding.

Lesson Eleven:
In-class end of the unit project
Students will be given a choice of the project that they wish to complete to summarize concepts they have learned throughout the unit. Choices will include: drawing a cartoon, creating a puzzle, writing and performing a song/rap, as well as, creating a website or powerpoint. In addition, students will be able to choose if they wish to work individually, in pairs, or in groups of 3. These will be presented to their classes.

Modifications/Optional

*****Students with preferential seating assignments according to their IEP are accommodated by front row seating. In addition, all notes and resources are provided on schoology for students to have printed if desired or more fitting to their learning plan.

*****Student(s) with hearing disabilities are given preferential seating and instruction is given face-to-face in order to provide the ability to read lips along with hear verbal directions/discussions.

*Students will be provided the opportunity to use notebooks, individual white boards or the class white board for various activities in class. Students are encourages to participate and work together to find an accurate solution, but more importantly, the most efficient procedure. Students will criticize each others work to determine the best strategies in the learning process.

Standards (imported)